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Senior School Certificate Examination

March 2017

Marking Scheme — Mathematics 65(B)

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/ Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65(B)

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Given equation can be written as $\dfrac{x-1/2}{\frac{\sqrt{3}}{2}} = \dfrac{y+2}{2} = \dfrac{z-3}{3}$ $\frac{1}{2}$

So, direction ratios of line parallel to AB are

$\dfrac{\sqrt{3}}{2}, 2, 3$ or $\sqrt{3}, 4, 6$ $\frac{1}{2}$

2. Derivative of $x^2 \cos x$ is $-x^2 \sin x + 2x \cos x$ 1

3. $y = mx + c \Rightarrow \dfrac{dy}{dx} = m$ $\frac{1}{2}$

$\Rightarrow \dfrac{d^2y}{dx^2} = 0$ $\frac{1}{2}$

4. $A' = \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix}$ $\frac{1}{2}$

$A - A' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\frac{1}{2}$

SECTION B

5. Let V be the volume of cube then $\dfrac{dv}{dt} = 9 \text{ cm}^3/\text{s}$

Surface area (S) of cube = $6x^2$, where x is the side

$V = x^3 \Rightarrow \dfrac{dv}{dt} = 3x^2 \dfrac{dx}{dt} \Rightarrow \dfrac{dx}{dt} = \dfrac{1}{3x^2}\dfrac{dv}{dt}$ 1

$S = 6x^2 \Rightarrow \dfrac{dS}{dt} = 12x\dfrac{dx}{dt} = 12x \cdot \dfrac{1}{3x^2}\dfrac{dv}{dt}$ $\frac{1}{2}$

$= \dfrac{4}{10} \times 9 = 3.6 \text{ cm}^2/\text{s}$ $\frac{1}{2}$

6. $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right) = \tan^{-1}\frac{2\sin^2 x/2}{2\sin x/2\cos x/2}$ $\frac{1}{2}+\frac{1}{2}$

$= \tan^{-1}\tan\frac{x}{2} = \frac{x}{2}$ $\frac{1}{2}$

So required derivative is $\frac{1}{2}$ $\frac{1}{2}$

7. $\text{adj}A = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}$ $\frac{1}{2}$

$|A| = -11$ $\frac{1}{2}$

LHS

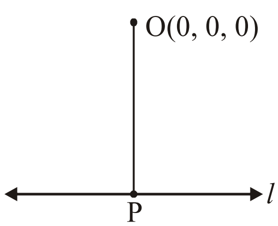
$\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$ $\frac{1}{2}$

RHS

$-11\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$ $\frac{1}{2}$

LHS = RHS hence verified.

8.



$\frac{x-4}{-1} = \frac{y-1}{3} = \frac{z-3}{-2} = \lambda(\text{say})$

Let P be foot of perpendicular from origin

So coordinates of P are $(-\lambda + 4, 3\lambda + 1, -2\lambda + 3)$ for some λ $\frac{1}{2}$

So, dr's of OP are $(-\lambda + 4, 3\lambda + 1, -2\lambda + 3)$

As $OP \perp l$ $\therefore (-\lambda + 4)(-1) + (3\lambda + 1)3 + (-2\lambda + 3)(-2) = 0$ $\frac{1}{2}$

Solving we get $\lambda = \frac{1}{2}$ $\frac{1}{2}$

$\therefore$ coordinate of P are $\left(\frac{7}{2}, \frac{5}{2}, 2\right)$ $\frac{1}{2}$

9. $f'(x) = 3x^2 - 6x + 9$ $\frac{1}{2}$

$= 3(x^2 - 2x + 3) = 3(x^2 - 2x + 1 + 2)$

$= 3[(x - 1)^2 + 2]$ 1

Clearly $f'(x) > 0 \ \forall x \in R$

$\therefore$ f(x) is strictly increasing on R $\frac{1}{2}$

10. $I = \int \frac{2}{2\left(x^2 + 3x + \frac{5}{2}\right)} dx$

$= \int \frac{dx}{\left(x + \frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$ 1

$= 2\tan^{-1}\left(\frac{x + 3/2}{1/2}\right) + C$ 1

or $2\tan^{-1}(2x + 3) + C$

11. Let number of tables be x

number of chairs be y

L.P.P. is maximize Z = 100x + 50y $\frac{1}{2}$

subject to constraints

$x + y \le 80$

$800x + 400y \le 50000$ $1\frac{1}{2}$

$x \ge 0,\ y \ge 0$

12. $P(A \cup B) = \frac{3}{5} \Rightarrow P(A) + P(B) - P(A \cap B) = \frac{3}{5}$ $\frac{1}{2}$

A & B are independent $\Rightarrow P(A) + P(B) - P(A) \cdot P(B) = \frac{3}{5}$ $\frac{1}{2}$

$\frac{1}{2} + p - \frac{p}{2} = \frac{3}{5}$ $\frac{1}{2}$

So $p = \frac{1}{5}$ $\frac{1}{2}$

SECTION C

13. Let $I = \int \frac{x^2 + x + 1}{(x-1)^3} dx$

Let $\frac{x^2 + x + 1}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$ 1

$\therefore I = \int \frac{1}{x-1} dx + 3\int \frac{1}{(x-1)^2} dx + 3\int \frac{dx}{(x-1)^3}$ $1\frac{1}{2}$

$= \log|x-1| - \frac{3}{x-1} - \frac{3}{2(x-1)^2} + c$ $1\frac{1}{2}$

14. $\Delta = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$ $\frac{1}{2}$

$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$ $\frac{1}{2}$

Applying $C_1 \leftrightarrow C_3$, then $C_2 \leftrightarrow C_3$

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (1 + xyz) = 0$ 1

$R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$

$$\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2 - x^2 \\ 0 & z-x & z^2 - x^2 \end{vmatrix} (1 + xyz) = 0$$

$(x - y)\ (y - z)\ (z - x)\ (1 + xyz) = 0$ 1

$\Rightarrow (x - y)\ (y - z)\ (z - x)\ (1 + xyz) = 0$

As x, y, z are unequal

$\therefore\ 1 + xyz = 0$ hence proved. 1

OR

$|A| = a\left(\frac{1+bc}{a}\right) - bc = 1 \neq 0$ $\frac{1}{2}$

$\text{adj}A = \begin{pmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{pmatrix}$ $\frac{1}{2}$

So $A^{-1} = \frac{\text{adj}A}{|A|} = \begin{pmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{pmatrix}$ 1

LHS	RHS	
aA^{-1}	$(a^2 + bc + 1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - a\begin{pmatrix} a & b \\ c & \frac{1+bc}{a} \end{pmatrix}$	
$a\begin{pmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{pmatrix}$	$\begin{pmatrix} a^2 + bc + 1 & 0 \\ 0 & a^2 + bc + 1 \end{pmatrix} - \begin{pmatrix} a^2 & ab \\ ac & 1+bc \end{pmatrix}$	
$\begin{pmatrix} 1+bc & -ab \\ -ac & a^2 \end{pmatrix}$	$\begin{pmatrix} bc+1 & -ab \\ -ac & a^2 \end{pmatrix}$	1+1
	LHS = RHS	

15. $\tan^{-1}\left(\dfrac{\frac{1}{4}+\frac{1}{6}}{1-\frac{1}{4}\cdot\frac{1}{6}}\right)+\tan^{-1}\dfrac{1}{x} = \dfrac{\pi}{4}$ 1

$\tan^{-1}\dfrac{10}{23}+\tan^{-1}\dfrac{1}{x} = \dfrac{\pi}{4}$ $\frac{1}{2}$

$\tan^{-1}\dfrac{1}{x} = \tan^{-1}1-\tan^{-1}\dfrac{10}{23}$ $\frac{1}{2}$

$\tan^{-1}\dfrac{1}{x} = \tan^{-1}\left(\dfrac{1-\frac{10}{23}}{1+1\cdot\frac{10}{23}}\right)$ 1

$\tan^{-1}\dfrac{1}{x} = \tan^{-1}\dfrac{13}{33}$

$x = \dfrac{33}{13}$ 1

16. $y = u + v$

$\dfrac{dy}{dx} = \dfrac{du}{dx}+\dfrac{dv}{dx}$ $\frac{1}{2}$

where $u = (\log x)^{2x}$, $v = (2x)^{\log x}$

$\log u = 2x \log(\log x)$, $\log v = \log x \cdot \log 2x$ 1

$\dfrac{1}{u}\dfrac{du}{dx} = \dfrac{2}{\log x}+2\log(\log x)$, $\dfrac{1}{v}\dfrac{dv}{dx} = \dfrac{\log 2x}{x}+\dfrac{\log x}{x}$

$\dfrac{du}{dx} = (\log x)^{2x}\left(\dfrac{2}{\log x}+2\log(\log x)\right)$, $\dfrac{dv}{dx} = (2x)^{\log x}\left(\dfrac{\log 2x+\log x}{x}\right)$ 1+1

$\therefore \dfrac{dy}{dx} = (\log x)^{2x}\left(\dfrac{2}{\log x}+2\log(\log x)\right)+(2x)^{\log x}\left(\dfrac{\log 2x+\log x}{x}\right)$ $\frac{1}{2}$

OR

$$\frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta) = a\theta\cos\theta$$ 1

$$\frac{dy}{d\theta} = a(\cos\theta + \theta\sin\theta - \cos\theta) = a\theta\sin\theta$$ 1

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$ $\frac{1}{2}$

$$\frac{d^2y}{dx^2} = \sec^2\theta\cdot\frac{d\theta}{dx} = \frac{\sec^3\theta}{a\theta}$$ 1

$$\left.\frac{d^2y}{dx^2}\right|_{\text{at }\theta=\pi/4} = \frac{8\sqrt{2}}{a\pi}$$ $\frac{1}{2}$

17. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y\cos\frac{y}{x} + x}{x\cos\frac{y}{x}} \qquad ...(A)$$ $\frac{1}{2}$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

(A) becomes

$$v + x\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v}$$

$$x\frac{dv}{dx} = \frac{1}{\cos v}$$

$$\int \cos v\, dv = \int\frac{dx}{x}$$ 1

$\sin v = \log|x| + c$ 1

$$\sin\frac{y}{x} = \log|x| + c$$ $\frac{1}{2}$

18. Writing $I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2}x}{\sin^{3/2}x + \cos^{3/2}x}dx \qquad ...(i)$ 1

Using property $x \to (\pi/6 + \pi/3 - x)$

$$I = \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2}x}{\cos^{3/2}x + \sin^{3/2}x}dx \qquad ...(ii)$$ 1

Adding

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x + \cos^{3/2} x}{\cos^{3/2} x + \sin^{3/2} x} dx$$

Integrating

$$2I = x \Big|_{\pi/6}^{\pi/3}$$ 1

$$I = \frac{\pi}{12}$$ 1

OR

$f(x) = x^2 + 1,\ a = 1,\ b = 3\quad nh = 2$ $\frac{1}{2}$

$$\int_1^3 (x^2+1)\,dx = \lim_{h\to 0} h[f(1)+f(1+h)+...+f(1+\overline{n-1}h)]$$ 1

$$= \lim_{h\to 0} h[2+(h^2+2h+2)+...+(n-1)^2 h^2 + 2(n-1)h + 2]$$ $\frac{1}{2}$

$$= \lim_{h\to 0} h[h^2 \Sigma(n-1)^2 + 2h\Sigma(n-1) + 2n]$$

$$= \lim_{h\to 0} h^3 \frac{(n-1)n(2n-1)}{6} + 2h^2 \frac{(n-1)n}{2} + 2nh$$

$$= \lim_{h\to 0} \frac{(nh-h)(nh)(2nh-h)}{6} + (nh-h)(nh) + 2nh$$ 1

$$= \frac{2\times 2\times 4}{6} + 2\times 2 + 4$$

$$= \frac{32}{3}$$ 1

19. $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} \Rightarrow \vec{a}\cdot(\vec{b}-\vec{c}) = 0\ \&\ \vec{a} \neq 0$

$\Rightarrow \quad \vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b}-\vec{c})$ $1\frac{1}{2}$

$\vec{a}\times\vec{b} = \vec{a}\times\vec{c} \Rightarrow \vec{a}\times(\vec{b}-\vec{c}) = \vec{0}$ and $\vec{a} \neq 0$

$\Rightarrow \quad \vec{b} = \vec{c}$ or $\vec{a}$ & $\vec{b}-\vec{c}$ are parallel $1\frac{1}{2}$

$\vec{a} \perp (\vec{b}-\vec{c})$ and $\vec{a} \parallel (\vec{b}-\vec{c})$ are not possible sumultaneously 1

So $\quad \vec{b} = \vec{c} \quad$ Hence proved.

20. $\vec{b}+\vec{c} = 2\hat{i}+3\hat{j}+2\hat{k}, |\vec{a}| = \sqrt{6}$ 1+1

Projection of $\vec{b}+\vec{c}$ on $\vec{a} = \dfrac{(\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}|}$

$= \dfrac{2+6+2}{\sqrt{6}}$ 1

$= \dfrac{10}{\sqrt{6}}$ or $\dfrac{5}{3}\sqrt{6}$ 1

21. Let number of units of type A = x $\frac{1}{2}$

number of units of type B = y

L.P.P. is Maximize profit Z = 10x + 8y 1

Subject to constraints

$\frac{3}{2}x + 3y \le 80$

$2x + y \le 70$ $1\frac{1}{2}$

$x, y \ge 0$

Value: Kindness or any other relevant value. 1

22. Let the events be

$E_1 \to$ Ball is drawn from Bag 1

$E_2 \to$ Ball is drawn from Bag 2 1

$A \to$ Ball drawn is Red

$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P(A/E_1) = \frac{3}{7}, P(A/E_2) = \frac{5}{11}$ 1

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{11}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{5}{11}}$$ 1

$$= \frac{35}{68}$$ 1

23. Let X denotes the number of aces

$p = \frac{1}{13}, \; q = \frac{12}{13}$ 1

X	P(x)	XP(x)
0	$\frac{144}{169}$	0
1	$\frac{24}{169}$	$\frac{24}{169}$
2	$\frac{1}{169}$	$\frac{2}{169}$

1+1

Mean $= 0 + \frac{24}{169} + \frac{2}{169} = \frac{2}{13}$ 1

SECTION D

24. $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix} \Rightarrow |A| = 10 \neq 0$ 1

$A_{11} = 4 \qquad A_{12} = 2 \qquad A_{13} = 2$

$A_{21} = -5 \qquad A_{22} = 0 \qquad A_{23} = 5$

$A_{31} = 1 \qquad A_{32} = -2 \qquad A_{33} = 3$ 2

$A^{-1} = \frac{\text{AdjA}}{|A|} = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$ $\frac{1}{2}$

Given system of equations can be written as

$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -2 \end{bmatrix}$ i.e. $AX = B \Rightarrow X = A^{-1}B$ 1

$X = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 8 \\ 4 \\ -2 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\therefore \; x = 1, \; y = 2, \; z = 3$ $1\frac{1}{2}$

25. For one-one

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$$

$$\Rightarrow x_1 \cancel{x_2} - 2x_1 - x_2 + \cancel{2} = x_1 \cancel{x_2} - x_1 - 2x_2 + \cancel{2}$$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

So f is one-one 2

For onto

Let $y \in B$

Let $f(x) = y$

i.e. $\frac{x - 1}{x - 2} = y$

$\Rightarrow x - 1 = xy - 2y$

$\Rightarrow x - xy = 1 - 2y$

$$\Rightarrow x = \frac{1 - 2y}{1 - y} \in A \ \forall\ y \in B$$

$\therefore$ f is onto 2

So f is invertible & $f^{-1}: B \rightarrow A$ defined as

$f^{-1}(x) = \frac{1 - 2x}{1 - x}$ 1

As $f^{-1}(x) = 7 \Rightarrow \frac{1 - 2x}{1 - x} = 7$

$\Rightarrow x = \frac{6}{5}$ 1

OR

Let $a, b \in R - \{-1\}$

$a * b = a + b + ab$

$b * a = b + a + ba$

$\therefore$ $a * b = b * a \; \forall \; a, b \in R \sim \{-1\}$

$\therefore$ $*$ is commutative 2

Let $a, b, c \in R - \{-1\}$

$a * (b * c) = a * (b + c + bc)$

$= a + b + c + bc + a(b + c + bc)$

$= a + b + c + bc + ab + ac + abc$

$(a * b) * c = (a + b + ab) * c$

$= a + b + ab + c + (a + b + ab)c$

$= a + b + ab + c + ac + bc + abc$

So $a * (b * c) = (a * b) * c$

$\therefore$ $*$ is associative 2

For identity

Let $e \in R - \{-1\}$ be identity element

$\therefore$ $a * e = a = e * a$

$a * e = a$

$\Rightarrow$ $a + e + ae = a$

$e(1 + a) = 0$

$e = 0$ as $1 + a \neq 0$

$\therefore$ 0 is the identity element 2

26. Let radius of circle be r

& side of square be a

So $2\pi r + 4a = k \Rightarrow a = \dfrac{k - 2\pi r}{4}$ 1

$A = \pi r^2 + a^2$

$$A = \pi r^2 + \frac{(k-2\pi r)^2}{16}$$ 1

$$\frac{dA}{dr} = 2\pi r - 4\pi \frac{(k-2\pi r)}{16}$$ 1

$$\frac{dA}{dr} = 0 \Rightarrow 2\pi r = \pi \frac{(k-2\pi r)}{4} \Rightarrow k = 8r + 2\pi r$$

So $a = \frac{8r + 2\pi r - 2\pi r}{4} = 2r$ $1\frac{1}{2}$

$$\frac{d^2A}{dr^2} = 2\pi + \frac{\pi^2}{2} > 0$$ 1

Thus area is minimum when a = 2r

i.e. side of square is double the radius of circle. $\frac{1}{2}$

27. Equation of required plane is

$$[\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4] + \lambda[\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5] = 0$$ 1

$$\vec{r}\cdot[(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}] = 4 - 5\lambda$$ 1

Above plane is perpendicular to $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k}) = -8$

$\therefore$ $(1 + 2\lambda)5 + (2 + \lambda)3 + (3 - \lambda)(-6) = 0$ 1

gives $\lambda = \frac{7}{19}$ 1

So required equation of plane is

$$\vec{r}\cdot\left(\frac{33}{19}\hat{i}+\frac{45}{19}\hat{j}+\frac{50}{19}\hat{k}\right) = \frac{41}{19}$$

or $\vec{r}\cdot(33\hat{i}+45\hat{j}+50\hat{k}) = 41$ 2

OR

Equation of line through (5, 1, 6) and (3, 4, 1)

is $\frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} = \lambda$ say 2 required point on line is

$(-2\lambda + 5,\ 3\lambda + 1,\ -5\lambda + 6)$ for some λ $1\frac{1}{2}$

As it lies on YZ plane

so $-2\lambda + 5 = 0 \Rightarrow \lambda = \frac{5}{2}$ 1

So required point is $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$ $1\frac{1}{2}$

28. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x^2}$$ $\frac{1}{2}$

Integrating factor is $e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$ 1

Solution is

$$y \cdot \log x = \int \frac{2}{x^2} \log x \, dx + c$$ 1

$$= \log x \left(\frac{-2}{x}\right) - \int \frac{1}{x}\left(\frac{-2}{x}\right) dx + c$$

$$y \cdot \log x = \frac{-2 \log x}{x} - \frac{2}{x} + c$$ 2

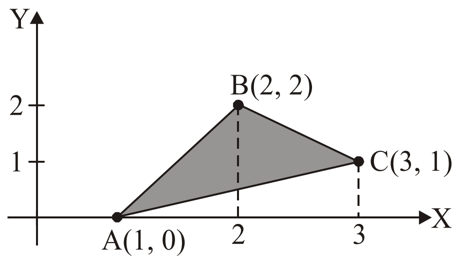
Putting $y = 0,\ x = 1$

$0 = 0 - 2 + c \Rightarrow c = 2$ 1

So particular solution is

$$y \cdot \log x = \frac{-2 \log x - 2}{x} + 2$$ $\frac{1}{2}$

29.



Equation of AB: $y = 2(x - 1)$

Equation of BC: $y = 4 - x$

Equation of AC: $y = \frac{x-1}{2}$ Correct equation of lines: $1\frac{1}{2}$

Correct figure 1

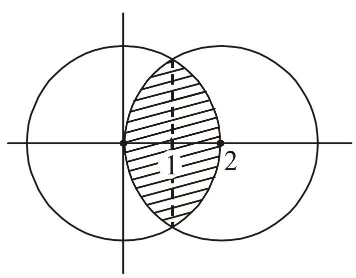
Required area $= \int_1^2 2(x-1)\,dx + \int_2^3 (4-x)\,dx - \int_1^3 \frac{(x-1)}{2}\,dx$ $1\frac{1}{2}$

$$= (x-1)^2\Big]_1^2 + \frac{(4-x)^2}{-2}\Big]_2^3 - \frac{(x-1)^2}{4}\Big|_1^3$$ 1

$$= 1 + \frac{3}{2} - 1$$

$$= \frac{3}{2} \text{ sq.units}$$ 1

OR



Correct figure 1

x coordinate of point of intersection is 1 1

Required area $= 2\left[\int_0^1 \sqrt{4-(x-2)^2}\,dx + \int_1^2 \sqrt{4-x^2}\,dx\right]$ 1

$$= 2\left[\frac{x-2}{2}\sqrt{4-(x-2)^2} + \frac{4}{2}\sin^{-1}\left(\frac{x-2}{2}\right)\Big|_0^1 + \frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\Big|_1^2\right]$$ 2

$$= 2\left[\frac{-\sqrt{3}}{2} - \frac{\pi}{3} + \pi + \pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right]$$

$$= \frac{8\pi}{3} - 2\sqrt{3} \text{ square units}$$ 1