KVS Junior Mathematics Olympiad (JMO) – 2005

M.M. 100 Time : 3 hours

Note : (i) Attempt all questions. Each question carries ten marks.

(ii) Please check that there are two printed pages and ten Questions in the question paper.

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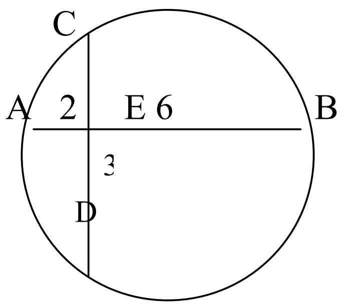
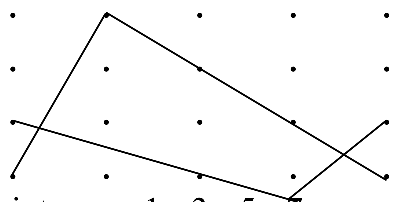
1. Fill in the blanks:
   (a) If four times the reciprocal of the circumference of a circle equals the diameter of the circle, then the area of the circle is …………..
   (b) If $1 - \frac{4}{x} + \frac{4}{x^2} = 0$ then $\frac{2}{x}$ equals………
   (c) If a=1000, b=100, c=10, and d=1, then (a+b+c-d) + (a+b-c+d) + (a-b+c+d)+(-a+b+c+d) is equal to …….
   (d) When the base of a triangle is increased by 10% and the altitude to the base is decreased by 10%, the change in area is ……….
   (e) If the sum of two numbers is 1, and their product is 1, then the sum of their cubes is ……………
2. (a) If $x = \left(\log_8^2\right)^{\log_2^8}$ find the value of $\log_3 x$.
   (b) If $\frac{4^x}{2^{x+y}} = 8$ and $\frac{9^{x+y}}{3^{5y}} = 243$ find the value of x-y.
3. (a) Find the number of digits in the number 2^{2005} x 5^{2000} when written in full.
   (b) Find the remainder when 2^{2005} is divided by 13.
4. (a) A polynomial p (x) leaves a remainder three when divided by x – 1 and a remainder five when divided by x-3. Find the remainder when p(x) is divided by (x-1) (x-3).
   (b) Find two numbers, both lying between 60 and 70, each of which is exactly divides 2^{43}-1.
5. In triangle ABC the medians AM and CN to the sides BC and AB, respectively intersect in the point O.P is the mid-point of side AC, and MP intersects CN in Q. If the area of triangle OMQ is 24 cm^2, find the area of triangle ABC.
6. The base of a pyramid is an equilateral triangle of side length 6 cm. The other edges of the pyramid are each of length $\sqrt{15}$ cm. Find the volume of the pyramid.
7. Chords AB and CD of a circle (see figure) intersect at E and are perpendicular to each other segments AE. EB and ED are of lengths 2cm, 6cm and 3cm respectively. Find the length of the diameter of the circle.



8. Three men A, B and C working together, do a job in 6 hours less time than A alone, in 1 hour less time than B alone, and in one half the time needed by C when working alone. How many hours will be needed by A and B working together, to do the job ?

9. Pegs are put on a board 1 unit apart both horizontally and vertically. A rubber band is stretched over 4 pegs as shown in the figure forming a quadrilateral. Find the area of the quadrilateral in square units.



10. The odd positive integers 1, 3, 5, 7 ….. are arranged in five columns continuing with the pattern shown on the right. Counting from the left, in which column (I, II, III, IV or V) does the number 2005 appear ? (Justify your answer)

I	II	III	IV	V
	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	33	35	37	39
47	45	43	41	
	49	51	53	55
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Junior Mathematics Olympiad - 2006

M.M.100

Time 3 Hours

NOTE: Attempt all questions, all questions carry equal marks.

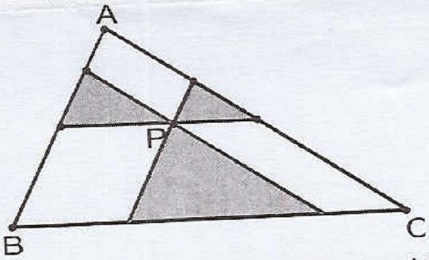
1. a, b, c are three distinct real numbers and there are real numbers x, y such that $a^3 + ax + y = 0$, $b^3 + bx + y = 0$ and $c^3 + cx + y = 0$. Show that $a + b + c = 0$.

2. The triangle ABC has CA = CB. P is a point on the circumcircle between A and B (and on the opposite side of the line AB to C). D is the foot of the perpendicular from C to PB. Show that $PA + PB = 2 \cdot PD$.

3. Given reals x, y with $(x^2 + y^2)/(x^2 - y^2) + (x^2 - y^2)/(x^2 + y^2) = k$, find $(x^8 + y^8)/(x^8 - y^8) + (x^8 - y^8)/(x^8 + y^8)$ in terms of k.

4. In a right triangle ABC right angled at B, a point P is taken on the side AB such that AP = h and PB = b. If BC = d and AC = y such that $h + y = b + d$. Prove that $h = bd/(2b+d)$

5. P is a point inside the triangle ABC. Lines are drawn through P Parallel to the sides of the triangle. The areas of the three resulting triangles with a vertex at P, have areas 4, 9 and 49. What is the area of triangle ABC?



6. A lotus plant in a pool of water is ½ cubit above water level. When propelled by air, the lotus sinks in the pool 2 cubits away from its position. Find the depth of water in the pool.

7. Let C_1 be any point on side AB of a triangle ABC. Join C_1C. The lines through A and B parallel to CC_1 meet BC and AC produced at A_1 and B_1 respectively. Prove that $1/AA_1 + 1/BB_1 = 1/CC_1$

8. The triangle ABC has angle B = 90°. When it is rotated about AB it gives a cone of volume 1920π □. Find the length AC. When it is rotated about BC it gives a cone of volume 800π □.

9. A number when divided by 7, 11 and 13 (the prime factor of 1001) successively leave the remainders 6, 10 and 12 respectively. Find the remainder if the number is divided by 1001.

10. Two candles of the same height are lighted together. First one gets burnt up completely in 3 hours while the second in 4 hours. At what point of time, the length of second candle will be double the length of the first candle.

10th KVS Junior Mathematics Olympiad (JMO) – 2007

M.M. 100 Time : 3 hours

Note : Attempt all questions.

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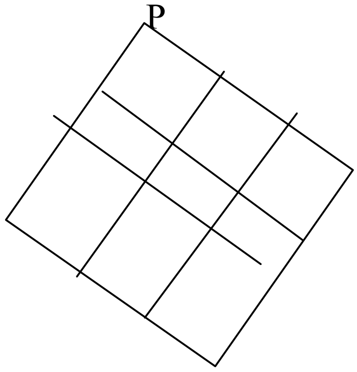
1. Solve
   $| x-1 | + | x | + | x + 1 | = x + 2$
2. Find the greatest number of four digits which when divided by 3, 5, 7, 9 leaves remainders 1, 3, 5, 7 respectively.
3. A printer numbers the pages of a book starting with 1. He uses 3189 digits in all. How many pages does the book have ?
4. ABCD is a parallelogram. P, Q, R and S are points on sides AB, BC, CD and DA respectively such that AP=DR. If the area of the parallelogram is 16 cm^2, find the area of the quadrilateral PQRS.
5. ABC is a right angle triangle with B = 90^o. M is the mid point of AC and BM = $\sqrt{117}$ cm. Sum of the lengths of sides AB and BC is 30 cm. Find the area of the triangle ABC.
6. Solve :
   $$\frac{\sqrt{(a+x)}+\sqrt{(a-x)}}{\sqrt{(a+x)}+\sqrt{(a-x)}} = \frac{a}{x}$$
7. Without actually calculating, find which is greater :
   31^{11} or 17^{14}
8. Show that there do not exist any distinct natural numbers a, b, c, d such that
   $a^3 + b^3 = c^3 + d^3$ and $a + b = c + d$
9. Find the largest prime factor of :
   $3^{12} + 2^{12} – 2.6^6$
10. If only downward motion along lines is allowed, what is the total number of paths from point P to point Q in the figure below ?

P



10^{th} KVS Junior Mathematics Olympiad (JMO) – 2007

M.M. 100 Time : 3 hours

Note : Attempt all questions.

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1. Solve
   $|x-1| + |x| + |x + 1| = x + 2$
2. Find the greatest number of four digits which when divided by 3, 5, 7, 9 leaves remainders 1, 3, 5, 7 respectively.
3. A printer numbers the pages of a book starting with 1. He uses 3189 digits in all. How many pages does the book have ?
4. ABCD is a parallelogram. P, Q, R and S are points on sides AB, BC, CD and DA respectively such that AP=DR. If the area of the parallelogram is 16 cm^2, find the area of the quadrilateral PQRS.
5. ABC is a right angle triangle with $B = 90^o$. M is the mid point of AC and BM = $\sqrt{117}$ cm. Sum of the lengths of sides AB and BC is 30 cm. Find the area of the triangle ABC.
6. Solve :
   $$\frac{\sqrt{(a+x)}+\sqrt{(a-x)}}{\sqrt{(a+x)}+\sqrt{(a-x)}} = \frac{a}{x}$$
7. Without actually calculating, find which is greater :
   31^{11} or 17^{14}

8. Show that there do not exist any distinct natural numbers a, b, c, d such that $a^3 + b^3 = c^3 + d^3$ and $a + b = c + d$
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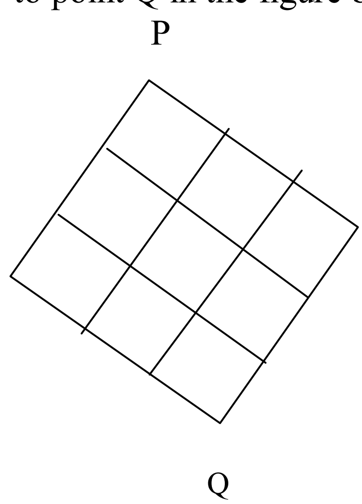

11^{th} KVS Maths Olympiad Contest – 2008

Time : 3 Hours M.M. : 100

NOTE: Attempt all questions. No electronic gadget is allowed during the examination.

1) Find the value of $S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots\ldots\ldots - 98^2 + 99^2$

2) Find the smallest multiple of ‘15’ such that each digit of the multiple is either ‘0’ or ‘8’.

3) At the end of year 2002. Ram was half as old as his grandfather. The sum of years in which they were born is 3854. What is the age of Ram at the end of year 2003?

4) Find the area of the largest square, which can be inscribed in a right angle triangle with legs ‘4’ and ‘8’ units.

5) In a Triangle the length of an altitude is 4 units and this altitude divides the opposite side in two parts in the ratio 1:8. Find the length of a segment parallel to altitude which bisects the area of the given triangle.

6) A number ‘X’ leaves the same remainder while dividing 5814, 5430, 5958. What is the largest possible value of ‘X’?

7) A sports meet was organized for four days. On each day, half of existing total medals and one more medal was awarded. Find the number of medals awarded on each day.

8) Let ΔABC be isosceles with $\angle ABC = \angle ACB = 78^0$. Let D and E be the points on sides AB and AC respectively such that $\angle BCD = 24^0$ and $\angle CBE = 51^0$. Find the angle $\angle BED$ and justify your result.

9) If α, β and γ are the roots of the equation.

$(x - a)(x - b)(x - c) + 1 = 0.$

Then show that a, b and c are the roots of the equation

$(\alpha - x)(\beta - x)(\gamma - x) + 1 = 0.$

10) A 4 x 4 x 4 wooden cube is painted so that one pair of opposite faces is blue, one pair green and one pair red. The cube is now sliced into 64 cubes of side 1 unit each.

(i) How many of the smaller cubes have no painted face?

(ii) How many of the smaller cubes have exactly one painted face?

(iii) How many of the smaller cubes have exactly two painted faces?

(iv) How many of the smaller cubes have exactly three painted faces?

(v) How many of the smaller cubes have exactly one face painted blue and one face painted green ?